9. Präsenzübung, **Statistische Physik**

zu bearbeiten am Donnerstag, 8.12.2011

Aufgabe P22 *Particle-antiparticle equilibrium*

The positron p is the antiparticle of the electron e. They have spin $\frac{1}{2}$ and the same mass M but opposite electric charge. It takes energy Δ to create an electron-positron pair from the vacuum, not counting their momenta. Ignoring any interaction between the particles, we want to express the expected concentration of electrons n as a function of Δ, M and the temperature τ.

a. Find n using the mass action law.

b. We will attempt to derive n also from first principles using the Boltzmann distribution of the whole system at temperature τ. Express the partition function (Boltzmann sum) Z for the whole system as a sum over the number of pairs, assuming that the particles are free in a box of volume $V = L^3$.

c. Assume that only terms in the sum with many pairs matter, and use the Stirling formula for $N!$ in order to show that

$$Z \simeq \cosh(4Ve^{-\Delta/2\tau} n_Q).$$

d. Use the partition function to compute n, in the limit $V \to \infty$.

Aufgabe P23 *Thermodynamics of the superconducting transition*

A metal in a superconducting state will revert to its normal conduction state if the temperature is higher than its critical temperature τ_c, or, alternatively, if an external magnetic field of magnitude larger than $B_c(\tau)$ is turned on at temperature $\tau < \tau_c$.
The free energy difference per unit volume between the normal and the superconducting phase of a superconductor satisfies

$$\frac{F_N(\tau) - F_S(\tau)}{V} = \frac{B_c^2(\tau)}{2\mu_0}$$

where μ_0 is the vacuum permeability (a constant of nature from the theory of electromagnetism.) At the transition temperature τ_c both sides become zero.

a. Compute the difference in entropy between the two phases as a function of the curve $B_c(\tau)$. Show that the energy U of both phases is equal at the transition temperature.

b. For $\tau < \tau_c$, the superconducting phase is that of lower entropy. There is no latent heat for the transition at $\tau = \tau_c$. What is the latent heat for the transition from the superconducting phase to the normal one when carried out in a magnetic field at $\tau < \tau_c$? (Computed so as to be positive.)

c. Express the difference $C_S - C_N$ of the heat capacity per unit volume in the superconducting and the normal phase. The heat capacity can be measured experimentally. It turns out that, at $\tau \ll \tau_c$, $C_S \ll C_N$. Using the "third law of thermodynamics" ($\sigma = 0$ at $\tau = 0$), show that, for $\tau \ll \tau_c$,

$$C_N \simeq -\frac{\tau}{\mu_0}\left(B_c \frac{d^2 B_c}{d\tau^2}\right)_{\tau=0}.$$